# EQUALITY OF OPPORTUNITY AND OPTIMAL CASH AND IN-KIND POLICIES

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#### Resumen

Este trabajo examina el argumento a favor de la provisión pública de un bien privado basado en la *igualdad de oportunidades*. Se estudia la frontera de posibilidades de utilidad de una sociedad compuesta por pobres y ricos donde existe preocupación por la distribución de un bien (ej. educación o salud), el que se asume no-suplementable. Se concluye que la política óptima depende crucialmente del grado de redistribución de la utilidad que es posible o deseable practicar.

#### Abstract

This paper examines the argument for public provision of a private good based on *equality of opportunity*. I study the utility possibility frontier of a society composed of poor and rich people in which there is a concern for the distribution of a good (e.g. education or health), which is assumed to be non-supplementable. The optimal policy depends on the degree of utility redistribution that a social planner wants or is able to perform.

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## 1. Introduction

Governments all around the world have a very active role in the provision of many essentially private goods. Prominent examples are education and health services. Although there is an extensive literature on the topic, the question of why there should be public provision is far from settled. Any strong argument intended to support public provision of education and health must offer a justification for the government to provide an in-kind transfer instead of cash. The normative public finance literature has dealt mainly with three major arguments: merit goods, redistribution, and market failure. Behind the concept of merit goods lies the idea of paternalism: the donor (usually the social planner) thinks that the recipient's preferences for education or health are a faulty representation of her well-being, so she will choose a "wrong" consumption bundle if given cash (Sandmo (1983), and Besley (1988)). The second argument lies in the distribution of the publicly provided good among individuals being socially preferred to the best implementable distribution of a cash transfer (Nichols and Zeckhauser (1982), Blackorby and Donaldson (1988), and Besley and Coate (1991)). Finally, the argument of market failure is mainly based on the existence of consumption externalities or informational problems, which make private markets inefficient or even prone to collapse (see Cohn and Geske (1990), and Barr (1992) for interesting surveys).

Although the above justifications are well known, in the real-world debate about the provision of education and health, policy makers, politicians, people in general and even economists usually prefer to invoke the argument of *equality of opportunity* (EO).<sup>1</sup> One of the versions of the EO argument roughly states that the government should provide basic education and health free of charge, or at subsidized prices, with the purpose of reducing dispersion in the consumption of those goods. To the extent that an individual's educational level and health state are considered to be beyond his/her control and are

closely related to future "opportunities", dispersion in these variables is viewed as unfair. Children do not choose the education quality they receive, yet take advantage of (or suffer) the consequences. Also, the attainable level of utility depends on the individual's health state which in many circumstances cannot be chosen. This paper is built upon the assumption that people care about the distribution of education and health, which I associate with a concern for equality of opportunity.<sup>2</sup> One can think of a society which cares about the starting point of the next generation, and wishes to equalize some basic factors that are beyond children's control, like education and health. For the sake of simplicity, the model presented here considers only one generation with good-specific distributional preferences, which can be seen as the reduced form of a more general problem.

Although the EO argument is generally recognized as one of the main justifications to sustain public schools or hospitals, its importance is not reflected in the current theoretical economic literature. This literature has repeatedly overlooked the argument by including it under one of the three categories mentioned above (merit goods, redistribution and market failures) with almost no discussion, and by ignoring it when analyzing optimal policies.<sup>3</sup> This paper takes one step to fill that gap by clarifying the public policy implications of people's concern about the distribution of certain specific goods, like basic education and health care. The paper is not only intended to specify the conditions under which such a concern implies a justification for free public provision, but also, and especially, to characterize how the cash versus in-kind dichotomy is decided upon along the utility possibility frontier, and which is the resulting social composition of people who attend public schools and hospitals.

The framework chosen for the analysis includes two features that are crucial for many results. (i) Education and health can be consumed for free in the public sector, but they can also be purchased in the private sector. (ii) Both goods are difficult to supplement: children cannot attend a public and a private school at the same time, and a health operation cannot be performed on a single person in two different hospitals.<sup>4</sup> These two points imply that people who look for a better education or health quality might opt-out of the public system, even if it is free.

The paper examines every point on the utility possibility frontier without inquiring about the mechanism which takes the economy from the laissez-faire situation to that point. Each of those points has associated with it an optimal combination of cash and inkind transfers. The in-kind transfer takes the form of free public provision of a given good to whoever wants to receive it. The discussion in the paper will be in terms of education, although basic health care should be kept in mind as well. Several elements should be taken into account to choose the optimal policy. (i) A cash transfer is always the "cheapest" way to attain a given target utility level for the poor since it does not distort behavior. (ii) However, a cash transfer is *target inefficient* since recipients ignore the externality caused by the concern about the distribution of education, an in-kind program *substitutes* for the entire private consumption, since education is assumed to be non supplementable. (iv) If the in-kind program is sufficiently large, it can attract not only the poor but also the rich, implying a much stronger effect on reducing dispersion than a cash transfer. The rest of the paper is organized as follows. The model is presented in section 2 and the basic results are explained in section 3. In section 4 some extensions are discussed, and section 5 concludes. All the proofs are presented in the Appendix.

### 2. The model

The economy is inhabited by a continuum of poor individuals (*P*) with mass equal to *N* who receive an exogenous income  $Y_p$ , and a continuum of rich people (*R*) with exactly the same mass *N* (just for the sake of simplicity) with income  $Y_r$  ( $Y_p < Y_r$ ). Both kind of individuals derive utility from consumption of a numéraire good (*x*) and education quality (*q*). Preferences over these two goods are given by the function U(x,q), which is assumed to be twice continuously differentiable, strictly quasi-concave and with positive marginal utilities  $U_1>0$  and  $U_2>0$ . Both goods are assumed to be non-inferior. In addition, rich people are assumed to have a concern for the distribution of education consumption. Such a concern is modeled as a function of the difference between the average education quality level of both groups. In particular, it is assumed that rich people's preferences are given by

$$U(x,q) - \frac{a}{2} \left(\overline{q}^r - \overline{q}^p\right)^2 \tag{1}$$

where  $\overline{q}^{i}$  is the average education quality level of group *i*, and  $\alpha \ge 0$  is a parameter that captures the relevance of the equity concern. For the sake of analytical simplicity it is assumed that poor people do not have an equity concern, so their utility function is just U(x,q).

Monetary incomes are modified by the action of the government, who can tax the rich and, either give the proceeds to the poor in the form of cash transfers, or fund a free public education program open to whoever wants to use it. I assume that institutional restrictions prevent taxing the poor or closing public schools to rich students.<sup>5</sup> Post-public-intervention nominal income (for simplicity, *post-tax income*) is denoted by  $y_i$ , *i=P,R* and defined as

$$y_r \equiv Y_r - C - E$$

$$y_p \equiv Y_p + C$$
(2)
(3)

where *C* are taxes paid by each rich person devoted to finance a cash transfer program, while *E* are taxes devoted to finance a free public education program. It is also assumed that taxes are never so high so as to make  $y_r$  lower than  $y_p$ . Education quality in the public sector ( $q_g$ ) is defined as

$$q_{g} \equiv \frac{E.N}{p.J} \tag{4}$$

where J is the mass of people who decide to attend public schools and p is the (constant) unit cost of quality. Notice that public education quality depends not only on the

educational budget but also on the number of people who decide to choose public provision. Education quality is assumed to be the same in all the public schools.<sup>6</sup>

Education services can also be purchased in a perfectly competitive market. With constant average costs equal to p, each unit of quality is sold at p. The optimal choice of q, provided that an individual decides to attend a private institution, is given by the demand function  $q^m(p, y_i)$  defined as:

$$q^{m}(p, y_{i}) \equiv \arg \max_{q} U(y_{i} - pq, q) \quad i = P, R$$
(5)

Notice that, even when equation (1) includes the dispersion of education quality as an argument of the rich's utility function, this term does not show up in equation (5): since there is an infinite number of rich people, the effect of one individual's decision over the global education quality dispersion is zero.

It is assumed that education quantity does not change: what does change is quality.<sup>7</sup> Also, consumption of education quality cannot be supplemented. People have to either buy it in the private sector, giving up the possibility of getting education for free in the public sector, or accept the quality offered in public schools, giving up the possibility of choosing its level.

Equation (6) defines  $q_i^{\prime}$  as the limit quality in public schools such that individual *i* is indifferent between public and private education.

$$U(y_{i} - pq^{m}(p, y_{i}), q^{m}(p, y_{i})) \equiv U(y_{i}, q_{i}^{l}) \quad i = P, R$$
(6)

When quality at public schools is lower than  $q_i^{\prime}$ , individual *i* prefers to opt-out of the public system school and attend a private institution to receive an education of a better quality.

Each person maximizes her utility by allocating her post-tax income between a numéraire x, whose price is normalized to 1, and education quality q. Individuals' equilibrium choices of education quality are summarized by:

$$q^{*}(p, y_{i}, q_{g}) = q_{g} \text{ if } q_{g} > q_{i}^{l}$$

$$= q^{m}(p, y_{i}) \text{ if } q_{g} < q_{i}^{l}$$

$$\in \{q_{g}, q^{m}(p, y_{i})\} \text{ if } q_{g} = q_{i}^{l}$$
(7)

The sequence of decisions in this model is as follows: (1) the government announces the educational budget and the cash transfer policy (and implicitly the tax policy and the post-tax incomes); (2) people make a prediction about public education quality  $q_g$ , which depends on the prediction of the number of students *J* who would choose public education; (3) having a guess about public school quality, people maximize utility and decide whether to go to a private school or to attend a public one. An equilibrium in this process is a situation where the number of students who decide to choose public education coincides with the guess about that number made in step (2).

## Equilibrium

Define G(J) as the mass of individuals who would choose public education if J were the mass of students they expect at public schools. To be more specific, define the following correspondence  $G: [0,2N] \rightarrow [0,2N]$ 

 $G(J) \in [N.I(J < J_r) + N.I(J < J_p), N.I(J \pounds J_r) + N.I(J \pounds J_p)]$ 

where  $J_{i=}(E.N)/(p.q_i)$ , and I(e) is unity if *e* is true and zero otherwise. Notice that if  $J=J_i$ , individual *i* is indifferent between attending a public or a private school. As *J* increases from that value, quality at public schools falls, and the individual will prefer the private option. Given a value *J*, the correspondence G(J) tells the number of individuals for whom  $q_i^I < q_g$ , plus the set of those for whom  $q_i^I = q_g$ .

The mass of students at public schools  $J^{\dagger}\hat{I}[0,2N]$  is defined as an equilibrium when everybody is maximizing utility given  $J^{\bullet}$ , and  $J \in G(J^{\bullet})$ . Notice that G(J) is an upper hemicontinuous correspondence that maps a nonempty, compact, convex set into itself, with the property that  $G(J) \in [0,2N]$  is nonempty and convex for every  $J \in [0,2N]$ . Therefore, Kakutani's fixed point theorem assures that G(J) has a fixed point, and hence that the equilibrium exists. Also, since the correspondence is always nonincreasing, the equilibrium is unique.

## Utility possibility frontier

In the framework presented above, I will study the shape and properties of the utility possibility frontier (UPF). Each point on the frontier shows the maximum utility that a typical rich agent can achieve, given an assured utility level for the representative poor agent. Associated to each point there is an optimal cash and in-kind policy. The frontier results from the following maximization problem:

$$\underset{C,E}{Max} L = V_{r}(C, E) + \mu(V_{p}(C, E) - V_{p}^{o})$$
(8)

where *L* is a Lagrangian which has to be maximized by choosing the appropriate levels of *C* and *E*, and  $V_i(C,E)$  is individual *i*'s indirect utility function which depends on the policy combination (*C*,*E*). The UPF is obtained by varying  $V_p^o$ , the *target* utility level for the poor, and plotting the resulting values of  $V_r$ .

# 3. Results

When there is no concern for the distribution of any particular good, the optimal public intervention is always through pure cash transfers. If the recipient's consumption pattern does not affect the donor's utility, the cheapest way to get the recipient to a given utility level is by giving him cash and letting him decide how to spend the money.<sup>8</sup> This result is so well-known in the literature that I prefer to skip the proof related to this framework. Notice also that, since there is no interdependence among agents, any transfer decreases the donor's utility so the UPF is negatively sloped everywhere.

Define  $V_p^s$  as the poor's utility level in the non-intervention allocation, i.e.  $V_p^s \equiv (Y_p - pq^m(p, Y_p), q^m(p, Y_p))$ . Notice that when the target utility level  $V_p^o$  is just the status-quo level  $V_p^s$ , the optimal policy is non-intervention. This result is in part trivial because it comes from the assumption that the poor cannot be taxed.<sup>9</sup>

For analytical simplicity, it will be assumed that the education quality demand is perfectly income inelastic in the rich's income range (i.e. preferences are locally quasilinear). This assumption is not crucial for the results. It is made just because it simplifies the analysis a great deal, especially due to the following implication: if rich's education demand is perfectly income-inelastic  $q_r^l$  does not depend on the value of  $y_r$ . Without this assumption, even a very small  $q_g$  could be attractive to a rich person if a heavy tax policy made her sufficiently poorer. Although in some cases this could be a realistic scenario, it introduces some complications without altering the main results of the paper.

Define  $V_p^{\ \prime}$  as the level of utility attainable by a poor person when offered a pure inkind transfer of quality  $q_r^{\ \prime}$  (i.e. the "limit" quality necessary to drive the rich to public schools), i.e.  $V_p^{\ \prime} \equiv (Y_p, q_r^{\ \prime})^{.10}$  It is easy to show that when the target level  $V_p^{\ \circ}$  is less than  $V_p^{\ \prime}$ , in equilibrium there are no rich people at public schools.<sup>11</sup>

Finally, define  $V_{\rho}^{e}$  as the poor's level of utility for which a pure in-kind transfer has exactly the same effect as a pure cash transfer of the same size ( $C^{e}$ ).<sup>12</sup>

 $V_{p}^{e} \equiv U(Y_{p} + C^{e} - pq^{m}(p, Y_{p} + C^{e}), q^{m}(p, Y_{p} + C^{e})) \equiv U(Y_{p}, C^{e}/p)$ 

## 3.1. Public education chosen only by the poor

This section investigates the optimal policies in the range where the rich are not attracted to public schools, i.e.  $V_p^{o} < V_p^{l}$ . The following proposition starts the analysis by characterizing the optimal policy for target utility levels  $V_p^{o} < V_p^{e}$ .

**Proposition 1**: For  $V_p^s < V_p^o < V_p^e$ , the optimal public intervention is through cash transfers.

Since the poor's indifference curves are strictly convex, the cheapest way to get to  $V_{\rho}^{o}$ from  $V_p^s$  is by a pure cash transfer. Moreover, for  $V_p^o < V_p^e$  that policy has the largest impact in reducing education quality dispersion. The key point is that while a pure cash transfer is used by poor people to supplement their consumption of education, any policy which includes an in-kind program substitutes for the poor's private consumption of education. When redistribution is "small" (i.e.  $V_p^{o} < V_p^{e}$ ), a small in-kind program is enough to get the poor to the target utility level. Hence, quality offered at public schools ends up being lower than the education guality bought by the poor under a pure cash transfer.<sup>13</sup> Thus, even when there is a specific concern about the distribution of education, when the utility redistribution is small, it should be performed via pure cash transfers. A sufficient condition for the optimality of such a policy when the rich are not attracted to public schools is  $q^{m}(p, Y_{p}+C) > C/p$ . From this condition, it is easy to see that the more sensitive the demand for education with respect to cash transfers (income), and the higher the education quality purchased by the poor when there is no public education, the "wider" the region where pure cash transfers are theoptimal policy (see Gasparini for a formal proof (1997)). When demand for education is highly sensitive to income in the poor's income range, a cash transfer can be very effective in increasing the poor's education quality, and thus reducing quality dispersion. Also, if the share of education in the poor's budget is large, it is necessary to spend a greater amount of resources to build a public education system which offers a quality level higher than that previously consumed by the poor. Again, the impossibility of supplementing public provision is crucial: even a small cash transfer can be used by the poor to buy more q in the market and supplement his consumption; instead, an in-kind program has to be big enough to entirely substitute the poor's private consumption of education.

Differentiation of (8) when cash transfers are the optimal policy yields the slope of the UPF in the region of  $V_{\rho}^{o} \in (V_{\rho}^{s}, V_{\rho}^{e})$ 

$$\frac{dV_r^*}{dV_p^o} = \frac{q_y^p.\alpha.(q^r - q^p) - U_1^r(.)}{U_1^p(.)}$$
(9)

The first term in the numerator is the gain for a rich person of giving \$1 when the rest of the rich also give \$1. That dollar increases the poor's consumption of education according to  $q_y^{\rho}$  (the change in the poor's education demand with respect to income), which in turn increases rich's utility by  $\alpha .(q^r - q^\rho)$ , where  $q^i = q^m(p, y_i)$  is the education quality bought by individual *i* in the private sector.<sup>14</sup> The second term is the rich's loss from giving up a dollar, i.e. her marginal utility of the numéraire consumption. The condition for a point in the UPF to be Pareto-dominated is  $q_y^{\rho} .a.(q^r - q^\rho) > U_1^{r}$ .

## Strong externality

An externality is defined as strong when rich people are better-off by all giving one dollar to finance education for the poor. The definition requires that this Pareto donation should be made under two conditions: (i) every rich gives one dollar, and (ii) resources should be fully allocated to finance education for the poor. Analytically, the condition for an externality to be strong is  $(1/p).a.(q^*(p,y_r,q_a)-q^*(p,y_p,q_a)) > U_1^r$ . From equation (9), if there is a strong externality and the poor's education demand is sufficiently sensitive with respect to income (and thus  $q_v^{\rho}$  is close to  $1/\rho$ ), there will be a region of Pareto-desirable redistribution. But if demand is relatively inelastic (small  $q_v^{p}$ ), even when there is a strong externality, there could be no allocations "close" to the laissez-faire (meaning in the range  $V_p^{s} < V_p^{o} < V_p^{e}$ ) which are Pareto superior to it. This point arises again from the fact that consumption of the good which causes the externality cannot be supplemented. From the status quo situation, if the rich build a "cheap" (low-quality) public education system, the poor might give up their private education and reduce their consumption of education quality. Instead, if the rich decide to give the equivalent of the education budget in cash, and the demand is sufficiently income-inelastic in the poor's income range, the increase in the poor's consumption of education will not be enough to make the rich better-off. Therefore, no policy can help the rich make a Pareto-desirable donation.

The next proposition studies the optimal public policy when the target utility level for the poor lies between  $V_{\rho}^{e}$  and  $V_{\rho}^{I}$ .

**Proposition 2**: Consider the range where  $V_p^o \in (V_p^e, V_p^l)$ .

(a) Pure cash transfers are never optimal.

(b) For  $V_p^{\circ}$  sufficiently close to  $V_p^{\circ}$ , a pure in-kind transfer is the optimal policy.

(c) If the externality is strong, a pure in-kind transfer is optimal.

(d) If the externality is not strong, a pure in-kind transfer could still be optimal, but this is more unlikely to occur as we move away from  $V_p^{e}$ .

To see why pure cash transfers are never optimal in the range being studied note that when  $V_{\rho}^{o} > V_{\rho}^{e}$  and rich people are not attracted to the public option, the outcome of a cash transfer can always be mimicked by some combination of cash and in-kind transfer of education. Also, note that when offered a pure cash transfer, the poor person is at a tangency point. Therefore, a small increase in public provision of education can be financed by a nearly equal reduction in the cash transfer keeping utility constant. The rich person will have a second order loss to keep the poor at the same utility level, but the larger educational budget will increase the poor individual's consumption of education quality, which is a first order gain for the rich person.

To see point (b) of proposition 2 recall that when the target utility level is equal to  $V_p^{e}$ , pure cash and pure in-kind policies are completely equivalent.<sup>15</sup> When we slightly increase the target utility from  $V_p^{e}$ , the cost to the rich of getting the poor to that level is similar under any pure policy. But this means that a pure in-kind transfer should be chosen, since it is the policy with the greatest effect on quality dispersion.

Regarding points (c) and (d) of proposition 2, note that an in-kind program is efficient to improve equity in education consumption (in the region of the UPF being analyzed) but it is more expensive to increase the poor's utility. However, when the externality is strong the rich do not care about the "price" of in-kind redistribution, since a dollar is worth more in the poor's hands. As soon as the externality becomes not strong the trade-off becomes effective. But a corner solution can still be optimal: reallocating money from public education to a cash transfer program reduces education quality received by the poor a great deal, since none of the poor's increased income will be allocated to buy more education (again because of non-supplementability). This effect can be larger than the rich's savings by using a cheaper tool for redistribution. As we move away from  $V_{\rho}^{e}$ , the dispersion in education quality, and hence the gains from reducing it, become smaller. Also, the distortion caused by a larger education program, and hence the gains by replacing part of it by a cash transfer program, become larger. Therefore, a combination of cash and in-kind transfers is likely to be the best policy.

Using the envelope theorem from (8) it can be proved that *if and only if the externality is strong, the slope of the utility possibility frontier is positive.* Also, from comparative statics it is easy to see that *an increase in the concern for the distribution of education (i.e. an increase in parameter a) leads to an increase in public provision of education, and to a decrease in cash transfers, given*  $V_p^{\circ}$ *. Overall, the amount of resources taken from the rich to finance public programs (cash and in-kind) increases* (see *Gasparini* (1997)). This is natural since an in-kind program is better for improving equality of opportunity (in the region of the frontier being analyzed). Hence, an increment in the rich's concern for the distribution of education leads to an increase in the size of the in-kind program. To keep the poor indifferent, the cash transfer must be reduced. Since cash transfers are "cheaper" than in-kind ones, rich's total contribution to public programs must increase.

## 3.2. Public education chosen by the poor and the rich

So far, the target utility level  $V_p^{o}$  was small enough so that any public education program that takes the poor from laissez faire to that target level was not attractive to rich people. When  $V_p^{o}$  reaches the value  $V_p^{\prime}$  a pure in-kind program needed to achieve that level offers a quality just enough to attract the rich. The number of rich people who choose public education depends on the value of *E*. In particular if  $E=Jpq_r^{l}/N$ , S=(J-N) rich people attend public schools while the rest attend private ones. Note that as *E* increases in the interval  $(p.q_r^{l}, 2p.q_r^{l})$ , the equilibrium public school quality level  $q_g$  does not move from the rich's limit quality  $q_r^{l}$ . A fixed quality with an increasing budget is explained by an increasing number of rich people who move to the public sector.

Rich people can be attracted to the public sector only when the target utility level for the poor is equal or greater than  $V_p^{l}$ . At that utility level, the relative power of in-kind provision over cash transfers increases discretely because it can be used to drive the rich to the public sector and hence, to obtain a more dramatic effect in reducing education quality dispersion. If this effect is strong enough, all taxes should be allocated to finance a public education program which attracts some of the rich. In *Gasparini* (1997) it is shown that even when the externality is not strong the optimal policy to reach  $V_p^{l}$  can be a pure inkind transfer attended by all the poor and some of the rich (i.e. a *semi-universal* program).<sup>16</sup> In the present paper I prefer to skip the discussion of that point and jump to the study of utility levels greater than  $V_p^{l}$  since the intuitions and results are basically the same.

There are several policies capable of achieving a target utility level larger than  $V_p^{l}$  (like  $V_p^{m}$  in Figure 1): (a) An in-kind program which offers  $q_g > q_r^{l}$  and attracts all the rich to public schools (*universal* in-kind program). This policy could be a pure in-kind program (point I in Figure 1), or an in-kind program combined with a cash transfer (a point on  $V_p^{m}$  between A and I, like U). (b) An in-kind program which offers  $q_g = q_r^{l}$  and drives just some of the rich to public schools (point A). (c) A combination of a cash and an in-kind program which offers  $q_g < q_r^{l}$  and is chosen only by poor people (a point on  $V_p^{m}$  between C and A, like B). (d) A pure cash transfer (point C).



I start by looking for the best universal in-kind program (labeled as  $q_u$ ), although this does not mean that it will necessarily be the best policy among all the policies mentioned above. Given that we are choosing among universal programs, equalization is complete under any policy. There are two forces which drive the choice of  $q_{\mu}$ : (i) It is convenient to choose a program that implies a quality level close to  $q^{\rho}$  because that level minimizes the transfer needed to achieve  $V_{p}^{o}$ , but also (ii) a quality level close to q' is desirable because at that point the rich's preferences are fully respected. To establish the range of variation for  $q_u$  one should find the optimal universal program that maximizes  $U(Y_r-C(q)-E(q),q)$ . Notice that in a universal program E(q)=2pq. Also, since we are moving along an indifference curve  $V_p^o$ ,  $C'(q) = -U_2^p/U_1^p$ . Therefore, the first order condition for an interior solution can be expressed as  $(U_2^r/U_1^r)-p=p-(U_2^p/U_1^p)$ , where marginal utilities are evaluated at points like U. Hence,  $q_u$  cannot be larger than q' because in that case the LHS would be negative and the RHS would be positive. Also it cannot be smaller than  $q^{\rho}$ because if it were, the LHS would be positive and the RHS negative, which again is a contradiction. Hence, the most preferred universal in-kind program will offer an education quality level always in the range between  $q^{\circ}$  and q'.

Once  $q_u$  is chosen it has to be compared with the rest of the policies which are capable of getting to  $V_p^{o} > V_p^{l}$ : a pure cash transfer, a reduced in-kind program, and a semi-universal in-kind program.

#### Universal public education versus pure cash transfers

In the Appendix it is shown that the condition for a universal public education program to be preferred to a pure cash transfer is

$$\frac{a}{2}(q^{r} - q^{p})^{2} > U_{1}^{r}(c).q_{c} + (q^{r} - q_{u})[U_{2}^{r}(u) - U_{1}^{r}(u).p]$$
(10)

where  $q_c = C_u + E_u/2 - C_c$ ,  $U_s^i(v)$  denotes marginal utility of good s for individual *i* evaluated at point v, and u and c refer to points like U and C in Figure 1. The LHS is the education quality dispersion under a pure cash transfer which is also the gain from switching to a universal in-kind program with complete equalization. The RHS shows two sources of savings when pure cash transfers are selected: (i) cash transfers are cheaper than any other policy to get the poor to any given target utility level.  $q_c$  is defined as the difference between the part of the costs associated to point U which are devoted to poor people (the cash transfer implicit in U plus half of the education expenditures) minus the cost of the pure cash program (allocated entirely to poor people). This value is necessarily nonnegative. (ii) A universal program induces the rich to buy a bundle different from the one they would have bought with a similar post-tax income but without subsidized education. The last term captures this distortion. When  $q_{\mu}$  is less than q' the term in brackets is positive and the whole term is positive. Notice that we did not have to worry about rich's preferences in last section since they chose their most preferred bundle at private markets. On the contrary, when  $q_{a} \ge q_{r}^{1}$  rich people are induced to consume public education, and thus, their decisions are distorted by the public policy.

An interesting case occurs if  $V_p^{\ \prime} < V_p^{\ \circ} < V_p^{\ e}$ . This situation is depicted in Figure 2. A pure cash transfer leads to point C and quality  $q^p$ , while a pure in-kind program is represented by point U and quality  $q_u$ . Since  $V_p^{\ \circ} < V_p^{\ e}$ , we know from last section, and it is

clear from the graph, that a pure cash transfer not only is the cheapest policy to get to  $V_{\rho}^{o}$ , but also it increases the poor's education quality consumption more than any other policy. In particular,  $q^{o} > q_{u}$ . That was the end of the story in last section since the rich were not attracted by quality  $q_{u}$ . However, when  $V_{p}^{o} > V_{p}^{\prime}$  as in the graph, a pure in-kind program necessarily becomes a universal program. In the comparison between U and C some new elements should be added. On the one hand, if U is chosen the rich's decisions are distorted (from  $q^r$  to  $q_u$ ); however, on the other hand, the effect on equity is more powerful. When  $V_p^{e} < V_p^{o} < V_p^{f}$  a pure in-kind program just moves the poor's education quality towards the rich's. If instead  $V_p^{\prime} < V_p^{\circ} < V_p^{\circ}$  education quality is completely equalized by driving both the rich and the poor to public schools. The condition for U to be preferred to C in Figure 2 is similar to inequality (10). This analysis leads us to conclude that if the concern for equality of opportunity is sufficiently large, a universal in-kind program could be better than a pure cash transfer, even when the latter leads to a higher education quality for every individual than the former. Of course, this result depends on the specific form of the externality assumed in this paper, which focuses on the concern about the distribution of education (or health). In a traditional externality where rich people care only about the poor's consumption a cash transfer will be unambiguously chosen in the above situation. Notice also the important difference with the redistributive arguments of public provision based on informational problems. In such justifications, public provision is optimal only when it does not attract the rich, so only reduced public programs make sense. In the case we just analyzed, if  $\alpha$  is sufficiently large the conclusion is quite different: it is by attracting the rich and becoming a *universal* program that public provision may be optimal.



## Universal public education versus public education only for the poor

When  $q^{\rho}$  is larger than  $q_r^{l}$ , a pure cash transfer dominates any policy which includes a *reduced* public education program chosen only by the poor because it implies both a lower cost and a lower level of quality dispersion. Hence, only the case where  $q^{\rho}$  is smaller than  $q_r^{l}$  will be considered. In the Appendix it is shown that the condition for a universal public education program to be preferred to a reduced public education program chosen only by the poor is:

$$\frac{a}{2}(q^{r} - q_{b})^{2} > U_{1}^{r}(b).q_{b} + (q^{r} - q_{u})[U_{2}^{r}(u) - U_{1}^{r}(u).p]$$
(10)

where  $q_b \equiv C_u + E_u/2 - C_b - E_b$ , and *b* refers to reduced programs like B in Figure 1. This expression is similar to the pure-cash one. The value of  $q_b$  is still positive since U is to the right of B which implies a higher distortion from  $q^b$ . The advantages of a reduced program are that it is cheaper and the rich's decisions are not distorted. However, if the concern for equality of opportunity is important and the distance between  $q^r$  and  $q_b$  is large, a universal program should be chosen.

#### Universal public education versus semi-universal public education

Recall that to attract only some of the rich to public schools,  $q_g$  should be equal to  $q'_r$ . Equation (12) is an approximation of the condition for a universal program to be preferred to a semi-universal program (see Appendix).

$$\frac{a}{2}((1-\frac{S}{N})(q^{r}-q_{r}^{l}))^{2} > U_{1}^{r}(a) q_{a} + [(1-\frac{S}{N})(q^{r}-q_{u}) + \frac{S}{N}(q_{r}^{l}-q_{u})] \cdot (U_{2}^{r}(u) - U_{1}^{r}(u).p)$$
(11)

where  $q_a = C_u + (E_u/2) - C_a - E_a(N/N+S)$ , and *a* refers to a semi-universal program (point A in Figure 1).

(i) A universal program achieves complete education quality equalization while in a semiuniversal one, some rich people attend private schools of a higher quality and thus, equalization is not complete. This effect is reflected in the LHS of (12).

(ii) As  $V_p^{o}$  increases and  $q^{o}$  moves to the right of  $q_r^{l}$ , a universal program becomes increasingly cheaper with respect to a semi-universal one in regards to the amount of resources devoted to poor people. The first term in the RHS captures this effect.  $q_a$  is the difference in resources devoted to the poor under both regimes. This difference tends to be negative as  $V_p^{o}$  and then  $q^{o}$  increase. The reason is that the optimal public school quality under a universal program  $(q_u)$  increases, while public school quality under a semi-universal program  $(q_r^{l})$  remains fixed.

(iii) Everybody in a universal program gets  $q_u$  while in a semi-universal one, some rich people (*S/N*) consume  $q_r^{l}$  and some others (1-*S/N*) buy  $q^{r}$ . Recall that  $q_r^{l} < q_u < q^{r}$ . Therefore, on the one hand, a universal program is good since it takes *S/N* rich people to a more desired bundle (from  $q_r^{l}$  to  $q_u$ ), but on the other hand it distorts the decision of (1-*S/N*) rich who reduce their education quality consumption from  $q_r$  to  $q_u$ . This effect is reflected in the second term of the RHS of (12), where  $(U_2^{r}-U_1^{r}p)$  is evaluated at  $q_u$ , and thus, it is positive.

Summing up, an in-kind transfer taken by the whole population has the property of achieving complete education equalization, which could make it preferable to any other policy combination if the concern for equalization is sufficiently large. In some circumstances, it could also have the advantage over non-universal programs of achieving  $V_p^o$  at a smaller cost. Finally, it implies a higher level of quality for some of the rich (i.e. a level closer to their most desired bundle) than a semi-universal program. Proposition 3 summarizes these findings.

**Proposition 3**: If the concern for equality of opportunity is sufficiently high, a universal inkind program (likely complemented with cash transfers) is optimal to achieve a level  $V_{\rho}^{o} > V_{\rho}^{I}$ .

The following proposition asserts that even when there is universal public education, and hence an increase in the educational budget does not reduce education quality dispersion, still it might be optimal to increase that budget.

**Proposition 4**: In the region where a universal in-kind program is the optimal policy an increase in  $V_{\rho}^{\circ}$  might lead to an increase in the size of that program.

To help understand the intuition behind this result, start from point U in Figure 1 and increase the poor's utility from  $V_p^m$  to  $V_p^n$ . To have a clearer intuition, assume that the slope of the poor's indifference curve at V (a point where the in-kind program does not grow) is equal to -p. This means that the resources transferred from the rich to the poor at point V are about the same as at point W, a point close to V but with a larger educational budget. The difference in the rich's post-tax income between those two points is equal to  $p.(q_v-q_w)$ . But rich people are getting more quality at W, more precisely the difference  $(q_w-q_v)$ . This means that by moving from V to W, rich people are "allowed" to buy education quality at a price p. Note that rich people are better-off with this deal, since they were consuming a lower education quality than desired at V (i.e.  $q_v=q_u < q^r$ ).

# Efficiency costs of public intervention

So far, it has been implicitly assumed that there are no costs associated with public intervention. To introduce costly intervention in the simplest possible way it is assumed that from each dollar paid by the rich there is only a fraction 0 < B < 1 available to support a cash or an in-kind program (or a combination of both). Equation (3) changes to  $y_p = Y_p + BC$  and (4) to  $q_g = BEN/pJ$ . These modifications do not substantially alter any previous result. In contrast, the existence of efficiency costs of public intervention is crucial as we move to higher levels of the poor's target utility  $V_p^{o}$ .

**Proposition 5**: If  $0 < \beta < 1$ , when  $V_p^{\circ}$  is sufficiently large a pure cash transfer is the optimal policy.

To see the intuition behind this proposition compare a pure cash transfer with a universal program. The former will be the preferred policy when (see Appendix):

$$\frac{\alpha}{2}(q^{r}-q^{p})^{2} < U_{1}^{r}(c).\theta_{c} + (q^{r}-q_{u})[U_{2}^{r}(u)-U_{1}^{r}(u).\frac{p}{\beta}] + U_{1}^{r}(u).p.q^{r}.(\frac{1}{\beta}-1)$$
(13)

As the target utility level  $V_{\rho}^{o}$  is increased,  $q^{\rho}$  grows and quality dispersion shrinks, making the LHS increasingly smaller. Also, as  $V_{\rho}^{o}$  goes up and  $q^{\rho}$  tends to  $q^{r}$ ,  $q_{u}$  tends to be equal to both. Hence, the first two terms in (13) tend to zero. But notice that the third term does not vanish as  $V_{\rho}^{o}$  grows large. Rather, it becomes larger as the rich's marginal utility of income increases and pq'((1/B)-1), which is positive, remains fixed. Hence, there will be a sufficiently large  $V_{\rho}^{o}$  such that (13) holds. The intuition is simple: when the target utility level is large, a cash transfer leads the poor to buy an education quality level in the private sector similar to the rich's. The effects of a pure cash transfer become similar to a universal program. But a difference remains: an in-kind program makes everybody "buy" education through the public sector rather than in the market, and thus, it generates an unnecessary efficiency loss.

A pure cash transfer is also better than a combination of cash and a reduced inkind program. As  $V_p^{o}$  increases, eventually we get to a point where  $q^{o}$  is greater than  $q_r^{l}$ (i.e. C is to the right of A). From that point on, a pure cash transfer implies less education quality dispersion than a reduced in-kind program, and is cheaper as well. Finally, for large values of  $V_p^{o}$  a pure cash transfer is better than an education program chosen by the poor and just some of the rich. In addition to the "*B*-inefficiencies", such a program needs to keep quality fixed at  $q_r^{l}$  and hence, it implies a higher education quality dispersion, a larger cost to achieve  $V_p^{o}$ , and a distortion on the behavior of some of the rich.

#### 3.3. Summing up: optimal policies along the UPF

The previous analysis has shown that optimal policies depend on many parameters. Two of them are especially relevant: the degree of concern about the distribution of education quality, and the target utility level for the poor. It might be clarifying to draw a graph showing the policy choices given different values of  $\alpha$  and  $V_{\rho}^{o}$ . To draw such a graph we need to make some assumptions with respect to the values of  $V_p^e$ ,  $V_p^{\prime}$  and  $V_p^c$ , where  $V_p^c$  is defined as the poor's utility level such that a pure cash transfer that leads the poor to that level implies an education quality bought by the poor of  $q_r^{\prime}$ . For simplicity only the case where  $V_p^{e} < V_p^{l} < V_p^{c}$  is presented. The resulting optimal-policy map is shown in Figure 3. The best policy is non-intervention on the vertical axis and a pure cash transfer on the horizontal axis. Pure cash transfers are also chosen for  $V_p^{o}$  less than  $V_p^{e}$  regardless of the value of  $\alpha$ . For values greater but close to  $V_p^{e}$ , a pure reduced in-kind program should be selected for any  $\alpha$ . This kind of program should be complemented with cash as  $V_{\rho}^{o}$ increases, unless  $\alpha$  is sufficiently large to make the externality strong or nearly strong. When we reach  $V_{p}^{\prime}$ , a semi-universal in-kind program is likely to be chosen, even when the externality is not strong. As we increase the target utility from  $V_p^{\ \prime}$ , a universal program can be optimal if the concern for equality of opportunity is important. From the level  $V_{\rho}^{c}$  on, reduced programs are dominated by pure cash transfers, and also semi-universal programs start to lose strength against universal ones. Finally, if public intervention inefficiencies are assumed, a pure cash transfer prevails when  $V_p^{o}$  is large enough.

Perhaps one of the main conclusions of the analysis is that given a certain concern for the distribution of education quality, any policy can be optimal depending on the degree of utility redistribution. Take the value  $\alpha_1$  in Figure 3. That value is consistent with pure cash transfers, pure in-kind programs, or a combination of both; and with public provision taken only by the poor, also by some rich people, or even by the whole population. If social planners followed the prescriptions of this analysis, we would not expect in-kind programs in societies with little utility redistribution. But we would expect the same observation from highly redistributive societies, even when the concern for equality of opportunity is high. Societies with a high concern for equality of opportunity are naturally more likely to have universal in-kind programs.

**Figure 3** The optimal-policy map



## 4. Some extensions

The implications of this model may be examined in some directions. This section includes a preliminary analysis of costly take-up, and taxes on the poor.

(1) It is important to recognize opportunity costs of attending school which make some people choose not to get any education at all, even when it is offered in the public sector for free. Analytically, opportunity costs can be captured by adding a function w(q) to the post-tax income:  $w(q)=w_o$  if q=0, and w(q)=0 if q>0. An individual *i* will be indifferent between attending a public school and not going to school at all when  $U(y_i+w(q_i),0) = U(y_i,q_g)$ . It is possible to show that opportunity costs are more burdensome for poor people, and therefore they are the first to drop school (see *Gasparini* (1995)). The introduction of costly take-up modifies some results of the previous analysis.

(i) Without costly take-up the optimal policy for the status-quo level  $V_p^s$  is nonintervention. The basic reason is that an in-kind transfer implies a lower quality level than the status-quo education quality bought by the poor in the market. Suppose that with opportunity costs and no public intervention poor people decide not to receive any education at all. Recall that, given that the poor cannot be taxed, any policy containing cash cannot be used if we want to stay at  $V_p^s$ . With costly take-up, a pure in-kind policy may drive the poor to school, and at the same time keep them at  $V_p^s$ . The way an in-kind program does so is by subtracting the value  $w_o$  from the poor's income. A pure in-kind program may be optimal but if the externality is not strong non-intervention is still the preferred policy.

(ii) In section 3.1. it was shown that for  $V_p^{o} < V_p^{e}$  a pure cash transfer is the optimal policy. However, notice that with opportunity costs the income effect of a pure cash transfer might not be enough to drive the poor to school. Thus, a pure in-kind program, although costly, might be chosen because it is helpful in reducing education quality dispersion.

(iii) When  $V_p^o > V_p^e$  (but less than  $V_p^h$ ) proposition 2 establishes that pure cash transfers should never be used. This conclusion may vary with costly take-up. An important point underlying proposition 2 is that a pure cash transfer can always be mimicked by a combination of a smaller cash transfer and some in-kind program. Now, consider the costly-take up case and assume  $w_o$  is big enough so as to lead the poor not to get an education when offered a given pure cash transfer. In that case the outcome of a pure cash transfer cannot be replicated anymore. The reason is that an in-kind transfer that drives the poor to public schools will make them lose the opportunity cost  $w_o$ . To take them back to the target level attained by a pure cash transfer and still want them at public schools requires a much larger budget than with a pure cash transfer. The key point is that an in-kind program has the additional cost of paying the cost  $w_o$  lost by poor people.

(iv) The introduction of costly take-up can offer a rationale for mandatory school. Suppose the optimal policy to attain  $V_{\rho}^{\circ}$  is a combination of cash and in-kind transfers, but that due to the presence of opportunity costs the poor decide not to attend school. Mandatory school can solve this problem as poor people are "forced" to move to the socially desired point on  $V_{\rho}^{\circ}$ .<sup>17</sup>

(2) When the possibility of taxing the poor is introduced some results change. Take for instance a target utility level equal to the status-quo situation, i.e.  $V_p^o = V_p^s$ . Ignoring the existence of opportunity costs of consuming education and with no taxes on the poor, the optimal policy is non-intervention. Notice that if the poor can be taxed and public policy is not costly, the non-intervention outcome can be replicated by taxing the poor and use the

proceeds to create a public education system that offers the same quality bought by the poor under non-intervention. But then, from that point it is optimal to increase the tax on the poor a little bit more and use the extra resources to increase the quality of public education. The marginal increase in taxes on the poor will not be enough to finance the increase in public education quality needed to place them back at  $V_p^{o}$ . However, the difference, that should be financed by the rich, is of a second order magnitude since the poor were at a tangency point. On the other hand, the increase in the quality of public education attended by poor people will decrease education dispersion, and hence will be a first-order gain for the rich.

The same line of reasoning can be applied to show that in the range  $V_p^{s} < V_p^{o} < V_p^{e}$  a pure cash transfer is not the optimal policy anymore when the poor can be taxed. A pure cash transfer financed by a tax on the rich can be replicated by a pure in-kind transfer financed by a similar tax on the rich and a tax on the poor. From that point, and using the same argument as above, it will be optimal to increase the size of the in-kind program.

## 5. Concluding remarks

Equality of opportunity is perhaps one of the most used arguments in the education and health debates. Yet there has been relatively little theoretical attention on what kind of public intervention it justifies. In our trip along the utility possibility frontier we have seen that there is not a unique policy to deal with equality of opportunity. In some regions of the UPF a pure cash transfer is optimal, in others in-kind provision or a combination of both instruments are the best policies. The choice of the optimal policy for a given utility redistribution depends on certain particular characteristics of the economy: income dispersion, income sensitiveness of the education quality demand, budget share of individual education expenses, and the degree of people's concern about education quality dispersion. The conclusions also depend on the existence of costs associated with consuming education that can lead to zero consumption, and on the possibility of taxing the poor.

Some of the conclusions for the case in which there are no costs of consuming education and no taxes on the poor are: (i) Even when people care about the distribution of education, if the desired utility redistribution is sufficiently small or sufficiently large, the best policy is a pure cash transfer. If utility redistribution is not set in any of those extremes, the use of cash transfers might still be optimal, although always combined with in-kind programs. (ii) Even when there is a strong externality involved in people's concern about equality of opportunity, sometimes there is no policy which can take the economy to a situation Pareto-preferred to the market allocation. (iii) For "middle" values of utility redistribution, public provision limited to the poor can be socially optimal. (iv) A public education (or health care) program attended by the whole population might be justified, even in a world with costly taxation and where cash transfers are possible, if the concern for equality of opportunity and the desired utility redistribution are sufficiently large. (v) The educational (or health) budget might be optimally increased beyond the point where it attracts the whole population and achieves complete equalization.

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# APPENDIX

## **Proof of proposition 1**

It is sufficient to prove that (i) a pure cash transfer requires fewer resources transferred from the rich to the poor to attain a level of utility  $V_{\rho}^{o}$  than any other policy, and (ii) a pure cash transfer results in a lower level of education quality dispersion than any other policy.

The first part is well-known and straightforward. Poor's indifference curves are strictly convex, thus, the cheapest way to get to a given level of  $V_p^o$  from the original poor's utility level is by a pure cash transfer (label it as  $C^o$ ).

For part (ii), label the education budget needed to get to  $V_p^o$  in a pure in-kind policy as  $E^o$ . Any combination of in-kind and cash which attains  $V_p^o$  implies  $E < E^o$  and then a lower level of education quality for the poor. So we need to compare only a pure cash transfer with a pure in-kind transfer and show that  $q^m(Y_p+C^o)>E^o/p$ . From the definition of  $V_p^e$  and strict convexity of indifference curves,  $q^m(Y_p+C^e) = C^e/p$ , and thus

 $Y_{p}+C^{e}-pq^{m}(Y_{p}+C^{e}) = Y_{p} \qquad (*)$ Since both goods are normal in the poor's income range and  $C^{o}<C^{e}$  $Y_{p}+C^{o}-pq^{m}(Y_{p}+C^{o}) < Y_{p}+C^{e}-pq^{m}(Y_{p}+C^{e})$ 

From (\*),

$$Y_{p}+C^{o}-pq^{m}(Y_{p}+C^{o}) < Y_{p}$$
 (\*\*)

From the definition of  $C^{\circ}$  and  $E^{\circ}$ 

$$U(Y_{p}+C^{\circ}-pq^{m}(Y_{p}+C^{\circ}),q^{m}(Y_{p}+C^{\circ})) \equiv V_{p}^{\circ} \equiv U(Y_{p},E^{\circ}/p)$$

Using (\*\*) and the fact that marginal utilities are positive we get that  $q^m(Y_p+C^\circ) > E^\circ/p$ .

## **Proof of proposition 2**

For part (a) assume by contradiction that a pure cash transfer is optimal for some  $V_p^{\circ}$  in the range  $(V_p^{\circ}, V_p^{\circ})$ . Thus,  $V_r(C,0) > V_r(x,y)$  for every *x* and *y* such that  $V_p(x,y) = V_p^{\circ} = V_p(C,0)$ . We can always choose a pair (C',E') such that  $V_p(C,0) = V_p(C',E')$  and C=C'+E'. Now reduce the cash transfer a little bit to *C''*, and increase *E* to *E''* so that  $V_p(C'',E'') = V_p(C,0)$ . We want to show that  $V_r(C'',E'') - V(C,0) > 0$ . This expression can be approximated by  $U_1^{\circ}$ .  $(C'+E'-C''-E'') + (\alpha/2).[(q'-E'/p)^2 - (q'-E''/p)^2]$ 

Since the poor are at a tangency point when the policy is (C',E'), (C'+E'-C''-E'') is negligible. Finally, the second term is positive since E''>E'.

For (b), (c) and (d) define *D* as the difference in the rich's utility between a pure in-kind program *E* and any combination of cash and in-kind transfers (*C'*,*E'*) which take the poor to  $V_{\rho}^{o}$ . *D* can be approximated by

$$D \approx (E - E').(\frac{1}{p}.\alpha.(q^{r} - \frac{E}{p}) - U_{1}^{r}) + C'.U_{1}^{r}$$

Consider a combination (*C'*,*E'*) sufficiently close to (0,*E*) (this is enough for our purposes). Then *E*-*E'* can be approximated by *C'*.*p*. $U_1^p/U_2^p$  and,

$$D \approx C' \left[ U_1^r + \frac{p \cdot U_1^p}{U_2^p} \cdot \left( \frac{1}{p} \cdot \alpha \cdot (q^r - E / p) - U_1^r \right) \right]$$

For (b), note that when  $V_p^o$  tends to  $V_p^e$ ,  $p.U_1^p/U_2^p$  tends to 1. Thus, since q' > E/p, *D* is positive and a pure in-kind transfer is the optimal policy. For (c) recall that a strong externality means  $(1/p).a.(q'-E/p) > U_1^r$ , which implies D > 0. For (d) notice that if  $(1/p).a.(q'-E/p) < U_1^r$ , *D* could still be positive if  $p.U_1^p/U_2^p$  is close to 1. This does occur close to  $V_p^e$ . But as we move away from that utility level,  $p.U_1^p/U_2^p$  (evaluated at a pure-in-kind-policy point) grows larger and the difference between quality levels gets smaller, so *D* tends to get smaller and eventually becomes negative.

#### **Proof of proposition 3**

This is based on Figure 1 where each policy is represented by a point on the poor's target utility level  $V_p^m$ . The sketch of this proof is divided in three parts:

(1) A universal in-kind program could be better than a pure cash transfer. Comparing the rich's utility levels under both policies, U is better than C if

$$\frac{a}{2} \cdot (q^{r} - q^{p})^{2} > U(Y_{r} - C_{c} - pq^{r}, q^{r}) - U(Y_{r} - C_{u} - E_{u}, q_{u})$$

where subscripts *u* and *c* refer to points U and C in Figure 1. Since  $C_c$  minimizes the transfer to the poor,  $q_c = C_u + (E_u/2) - C_c > 0$ . Also recall that  $E_u/2 = pq_u$ . Then the above equation can be rewritten as

$$\frac{a}{2} \cdot (q^{r} - q^{p})^{2} > U(Y_{r} - C_{c} - pq^{r}, q^{r}) - U(Y_{r} - C_{c} - q_{c} - pq_{u}, q_{u})$$

The last term can be decomposed to get

$$\frac{\alpha}{2} \cdot (q^{r} - q^{p})^{2} > U(Y_{r} - C_{c} - pq^{r}, q^{r}) - U(Y_{r} - C_{c} - \theta_{c} - pq^{r}, q^{r}) + U(Y_{r} - C_{c} - \theta_{c} - pq^{r}, q^{r}) - U(Y_{r} - C_{c} - \theta_{c} - pq_{u}, q_{u})$$

The first two terms of the RHS can be approximated by  $U_1^r(c).q_c > 0$ , and the last two by  $(q^r-q_u).(U_2^r(u)-U_1^r(u)p)$ , where  $U_s^i(v)$  denotes marginal utility of good *s* for individual *i* evaluated at point *v*. Applying these approximations to the above inequality leads to inequality (10) in the text which is positive for a sufficiently large  $\alpha$ .

(2) A universal in-kind program could be better than a reduced in-kind program. The condition for U to be better than B is

$$\frac{\alpha}{2} (q^{r} - q_{b})^{2} > U(Y_{r} - C_{b} - E_{b} - pq^{r}, q^{r}) - U(Y_{r} - C_{u} - E_{u}, q_{u})$$

where  $C_b$  and  $E_b$  are meant to belong to point B. Define  $q_b \equiv C_u + (E_u/2) - C_b - E_b > 0$ . A decomposition similar to point (1) can be performed to get equation (11) in the text which is positive for a large value of  $\alpha$ .

(3) A universal in-kind program could be better than a semi-universal program. Comparing the rich's utility at U and A we get that a universal in-kind program is better than a semi-universal one when

$$\frac{\alpha}{2} \cdot \left( (1 - \frac{S}{N}) \cdot (q^{r} - q_{r}^{l}) \right)^{2} > (1 - \frac{S}{N}) \cdot U(Y_{r} - C_{a} - E_{a} - pq^{r}, q^{r}) + \frac{S}{N} \cdot U(Y_{r} - C_{a} - E_{a}, q_{r}^{l}) - U(Y_{r} - C_{u} - E_{u}, q_{u})$$

where  $C_a$  and  $E_a$  are meant to belong to point A. At that point,  $q_r^{l}=E_aN/p(N+S)$  so  $E_a=pq_r^{l}+(S/N).pq_r^{l}$ . Define  $q_a\equiv C_u+(E_u/2)-C_a-E_a.(N/N+S)$  which, again, is the difference of resources devoted to the poor under both regimes. When  $V_p^{o}$  increases, this difference tends to be negative, as  $q^{o}$  and  $q_u$  move to the right while  $q_r^{l}$  remains fixed. Replacing  $q_a$ , performing a decomposition similar to points (1) and (2), and assuming constant marginal utility of the numéraire along  $V_p^{o}$  (this is just to present a clearer result) we get that equation (12) in the text is an approximation to the condition which assures that U is better than A. Equation (12) holds for a sufficiently large  $\alpha$ .

## **Proof of proposition 4**

When both kind of individuals choose public education, optimization problem (8) becomes:

Max 
$$L_{C,E} = U(Y_r - C - E, \frac{E}{2p}) + m[U(Y_p + C, \frac{E}{2p}) - V_p^o]$$

Note that  $dq_r^{l}/dy_r=0$  implies  $U_{12}^{r}=U_{11}^{r}U_{2}^{r}/U_{1}^{r}$ . Thus, comparative statics yield

$$\frac{dE}{dV_p^o} = \frac{1}{H} \cdot \left\{ \frac{U_{11}^r \cdot U_2^p}{2p} \left[ \frac{2p \cdot U_1^p}{U_2^p} \left( 1 - \frac{1}{2} \frac{U_2^r}{pU_1^r} \right) - 1 \right] + \frac{\mathbf{m}}{2p} \left( U_1^p \cdot U_{21}^p - U_2^p \cdot U_{11}^p \right) \right\}$$

where *H* is the corresponding (positive) hessian. To understand this expression, assume  $q_{\mu}=q^{\rho}$ , and thus  $p_{\nu}U_{1}^{\rho}=U_{2}^{\rho}$ . The above equation becomes

$$\frac{dE}{dV_p^o} = \frac{1}{H} \cdot \left\{ \frac{U_{11}^r \cdot U_2^p}{2p} (1 - \frac{U_2^r}{pU_1^r}) + \frac{\mathbf{m}}{2p} (U_1^p \cdot U_{21}^p - U_2^p \cdot U_{11}^p) \right\}$$

At  $q_u < q^r$ ,  $U_2^r > pU_1^r$ . Therefore, assuming decreasing marginal utility of consumption, the first term is positive. Since *q* is normal in the poor's income range the second term is positive as well. Hence the whole expression  $dE/dV_p^o$  is positive. If the universal program implies  $q_u, > q^o$  the first term is negative and the sign of the whole expression is ambiguous.

## Proof of proposition 5

The conditions under which a pure cash transfer is better than a universal program are studied in part 1 of proposition 3. If we introduce the inefficiency cost B, C is better than U when

$$\frac{\boldsymbol{a}}{2} \cdot (q^r - q^p)^2 < U(Y_r - C_c - pq^r, q^r) - U(Y_r - C_c - q_c - \frac{pq_u}{\boldsymbol{b}}, q_u)$$

A decomposition can be performed to get

$$\frac{\alpha}{2} \cdot (q^{r} - q^{p})^{2} < U(Y_{r} - C_{c} - pq^{r}, q^{r}) - U(Y_{r} - C_{c} - \theta_{c} - pq^{r}, q^{r}) + U(Y_{r} - C_{c} - \theta_{c} - pq^{r}, q^{r}) - U(Y_{r} - C_{c} - \theta_{c} - \frac{pq^{r}}{\beta}, q^{r}) + U(Y_{r} - C_{c} - \theta_{c} - \frac{pq^{r}}{\beta}, q^{r}) - U(Y_{r} - C_{c} - \theta_{c} - \frac{pq_{u}}{\beta}, q_{u})$$

The second term can be approximated by  $U_1^r p.q'.((1/\beta)-1)$  which is positive. Equation (13) is derived using the procedure of part 1 of proposition 3. The rest of the proof follows the lines given in the text.

<sup>2</sup> Social concern about the distribution of particular goods is also usually known as *specific egalitarianism* or *commodity egalitarianism* (see *Tobin* (1970)). The concept of equity as equality of opportunity has been treated in the economic and political philosophy literature. See for example *Coleman et al.* (1966), *Archibald and Donaldson* (1979), *Arneson* (1989), *Green* (1989), *LeGrand* (1991), *Gravel* (1994), *Roemer* (1997), and *Rubin* (1996).

<sup>3</sup> See, for example, *Barr* (1992) and *Poterba* (1994). This paper does not claim that the EO argument is a completely autonomous justification for public provision, but rather, that it should not be included into any of the mentioned categories without discussion because it has different features from the pure arguments.

<sup>&</sup>lt;sup>1</sup> To give one of many examples, the new Argentinean Constitution establishes that it is the authority of the National Congress "to make laws regarding the organization of education which...ensure...the *equality of opportunity* and guarantee the principles of equity and free of charge provision of public education". (*Constitución Nacional Argentina* (1994), article 75, clause 19).

<sup>&</sup>lt;sup>4</sup> In many cases there exists the possibility of supplementing consumption to some extent. However, the important point is that supplementation is costly. Sometimes that cost is endogenous. For instance, the government can offer public school for free, or can issue educational vouchers to be used in any school which can be

supplemented if desired. See *Blomquist and Christiansen* (1995) for a model with endogenous possibility to supplement public provision.

<sup>5</sup> The case in which the poor are also taxed is treated in the extensions section.

<sup>6</sup> We can think of institutional constraints which prevent the government from "discriminating" among its schools, and/or assume that people would move from worse to better public schools, and that such a process would continue up to the point of complete quality equalization among public schools.

<sup>7</sup> The possibility of choosing not to get an education is considered in the extensions section when opportunity costs of attending school are introduced.

<sup>8</sup> The gender of the rich and the poor was decided by a fair coin: male for the poor and female for the rich.

<sup>9</sup>Notice that if it were possible to tax the poor, it might be optimal to do so and use the proceeds (plus some additional resources provided by the rich) to fund a public school system of a higher quality than the private one attended by the poor in the laissez-faire situation (see section 4).

<sup>10</sup> Notice that  $V_p^{\ \prime} \ge V_p^{\ s}$ . Also, note that the assumption of an income-inelastic demand in the rich's income range allows us to define a unique  $V_p^{\ \prime}$  and makes the analysis significantly simpler.

<sup>11</sup> See the extended version of this paper for a proof (*Gasparini* (1997)).

<sup>12</sup> Notice that the ensuing definition implicitly assumes that rich people still choose private schools when they are offered the possibility to attend public schools with the quality implied by the equation below, i.e.  $V_p^{\ e} < V_p^{\ l}$ . The case where  $V_p^{\ e} > V_p^{\ l}$  will be analyzed in the next section.

<sup>13</sup> The difference between the two policies is even more extreme for values of  $V_{\rho}^{o}$  close to  $V_{\rho}^{s}$ : an in-kind program that places the poor at  $V_{\rho}^{o}$  might offer a *lower* quality than the quality bought in the market by the poor in the non-intervention situation. The possibility of a reduction in the level of education quality brought about by introducing subsidized public education is pointed out in *Peltzman* (1973), and documented in *Ganderton* (1992) for the college level.

<sup>14</sup> Notice that the rich's average education quality level can be replaced by a particular rich person's education quality level since all the rich behave in the same way at equilibrium. The same applies to the poor.

<sup>15</sup> Notice that any combination of these policies which take the poor to  $V_p^e$  would imply higher taxes on the rich (than the pure policies) and a lower education consumption by the poor, so it would never be chosen.

<sup>16</sup> A pure in-kind transfer that attains  $V_p^{\prime}$  is represented as point P in Figure 1. Point P is consistent with any value of *S*. A larger value implies a higher degree of education quality equalization, but also a larger educational budget and a corresponding heavier tax burden for the rich.

<sup>&</sup>lt;sup>17</sup> It is implicitly assumed that it is possible to force people to get education in the public sector, but it is not possible to force them to buy a particular quality in the market.